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II. "On the Stability of Domes."—Part II. By E. WYNDHAM TARN, M.A., Memb. Roy. Inst. Brit. Architects. Communicated by G. GODWIN, Esq. Received October 23, 1866.

(Abstract.)

In a former paper on this subject which the author presented to the Royal Society, and which is published in the 'Proceedings' (vol. xv. p. 182), he obtained formulæ for calculating the thrust of a spherical dome of uniform thickness, by supposing it to consist of a number of thin ribs, each of which is formed by two vertical planes intersecting at the axis of the dome, and making a small angle with each other; and then treating each rib as forming, with the corresponding one of the opposite side, a complete arch.

In the present paper the author applies the same method to domes of other forms than the spherical. The following are the kinds for which the formulæ are investigated:—

A. The Gothic dome, of which that of the cathedral at Florence is a splendid example. This kind has for its section a pointed arch formed by two segments of circles, the centre from which each is struck being on the springing line, but not in the axis of the dome. The author denotes by  $\alpha$  the angle between the vertical and a line drawn from this centre to the point, near the crown of the dome, where its axis is intersected by a circle drawn around that centre with a radius equal to the mean of the radii of the circles which generate the outer and inner surfaces of the dome, and reduces his results to numbers for three different values of the angle  $\alpha$ .

B. A dome whose inner surface is a paraboloid of revolution, the thickness of the shell being uniform throughout.

C. The dome whose surface is formed by the revolution of an ellipse about its major axis. In the investigation, the two surfaces are supposed to be generated by the revolution of two concentric elliptic quadrants, whose major axes differ from one another by the same quantity as the minor axes, so that the thickness is very nearly uniform throughout.

D. A form of dome commonly used in eastern countries, sometimes called the ogival dome, the surface of which is generated by the revolution of a curve which has a point of contrary flexure. In the investigation, the author takes for this curve the curve of sines, the equation of which is  $y' = r \sin \frac{x'}{r}$ , where  $x'$ ,  $y'$  are the vertical and horizontal coordinates of a point in the generating curve, and supposes the outer and inner surfaces to be generated by the revolution of two such curves, differing only by the value of  $r$ , the origin being the same.

The following Table exhibits the principal results obtained from the author's investigations. All the domes, except the last, are supposed to be of the uniform thickness throughout of 1 foot. In the last the thickness is 1 foot at the springing, but gets rather less towards the top. The domes are

supposed to be built of material weighing 125 lbs. to the cubic foot. The thrust is calculated for the 180th part of the whole dome, being the portion cut out by two planes which make an angle of  $2^\circ$  at the axis.

Table showing the position of the weakest joint in domes of various forms, and the horizontal thrust at that joint.

	Form of Dome.	Span.	Position of the weakest joint, or joint where thrust is greatest.	Greatest horizontal thrust, or thrust at weakest joint.
		feet.		lbs.
1.	Hemisphere .....	20	{ Makes with springing line an angle of $20^\circ$ .	92.01
2.	Gothic, $\alpha = 10^\circ$ .....	20	{ Makes with springing line an angle of $17^\circ$ .	88.7
3.	Gothic, $\alpha = 22\frac{1}{2}^\circ$ .....	20	{ Makes with springing line an angle of $13\frac{1}{2}^\circ$ .	80.418
4.	Gothic, $\alpha = 30^\circ$ .....	20	{ Makes with springing line an angle of $11^\circ$ .	77.27
5.	Parabolic; height above springing equals the half span.	20	At springing line .....	79.6
6.	Elliptical, major axis vertical; ratio of major to minor axis as 6:5.	20	{ One-third of semimajor axis above springing line.	90.867
7.	Ogival; contour, the "curve of sines."	20	{ One-sixteenth of the span above springing line.	62.5

In the preceding cases, except the last, the domes are assumed to be of uniform thickness. The author finally applies his formulæ to the case of the spherical dome in which the thickness at the crown is one-half that at the springing, the inner surface being generated by the revolution of a circular quadrant whose centre is raised above the centre of that which generates the outer surface by half the difference of the radii. Assuming the outer and inner radii  $R, r$  to be respectively 11 feet and 10 feet, he finds that the weakest point on a dome of this form appears to be at a height equal to  $\frac{4}{11}R$  above the springing line; and with the other numerical values, assumed the same as before, he finds for the horizontal thrust at the weakest joint 55.78 lbs.

For each kind of dome the author forms what he calls the *equation of stability*, giving, for an assumed value of the height of the pier, the least thickness  $t$  which will permit of stability. The following are the results for each kind of dome, the height of the pier being taken at 50 feet:—

Spherical dome (from former paper)	$t = 2.45$	feet.
Gothic dome ( $\alpha = 22\frac{1}{2}^\circ$ )	$t = 2.259$	"
Parabaloidal dome	$t = 2.244$	"
Elliptic dome	$t = 2.44$	"
Ogival dome	$t = 2$	"
Spherical dome (thinner at crown)	$t = 1.9$	"

The author considers what he has found as the weakest part of a dome to be the position in which an iron belt must be placed to produce the greatest effect in counteracting the thrust of the dome. If this be done, the thickness of the pier may be considerably diminished, and need not greatly exceed the strength necessary for supporting the superincumbent weight acting vertically downwards.

### III. "A Supplementary Memoir on Caustics."

By A. CAYLEY, F.R.S. Received November 15, 1866.

(Abstract.)

It is near the conclusion of my "Memoir on Caustics," *Phil. Trans.* vol. cxlvii. (1857), pp. 273, 312, remarked that for the case of parallel rays refracted at a circle, the ordinary construction for the secondary caustic cannot be made use of (the entire curve would in fact pass off to an infinite distance), and that the simplest course is to measure off the distance GQ from a line through the centre of the refracting circle perpendicular to the direction of the incident rays. The particular secondary caustic, or orthogonal trajectory of the refracted rays, obtained on the above supposition was shown to be a curve of the order 8; and it was further shown by consideration of the case (wherein the distance GQ is measured off from an arbitrary line perpendicular to the incident rays), that the general secondary caustic or orthogonal trajectory of the refracted rays was a curve of the same order 8. The last-mentioned curve in the case of reflexion, or for  $\mu = -1$ , degenerates into a curve of the order 6; and I propose in the present supplementary memoir to discuss this sextic curve; viz. the sextic curve which is the general secondary caustic or orthogonal trajectory of parallel rays reflected at a circle.

*November 30, 1866.*

#### ANNIVERSARY MEETING.

Lieut.-General SABINE, President, in the Chair.

Dr. Gladstone, on the part of the Auditors of the Treasurer's Accounts appointed by the Society, reported that the total receipts during the past year, including a balance of £15 9s. carried from the preceding year, amounted to £4295 16s. 11d.; and that the total expenditure in the same